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PROBLEMS OF HEAT CONDUCTION FOR AN ANGULAR REGION WITH AN INTERNAL SOURCE

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Exact solutions of nonstationary problems of heat conduction have been obtained for an unbounded rectangular region when the opening angle is equal to $\pi/(2n+1)$, where n is any natural number. By passage to the limit it has been shown that no stationary regime is possible for the rectangular region in the case of action of a constant internal source. The exact solution of the stationary problem for an angular region with an arbitrary opening angle κ_0 has been given. It has been proved that in the presence of a constant heat source the stationary regime is possible just for the acute angle $\kappa_0 \le \pi/2$, while for the right or obtuse angles $\kappa_0 \ge \pi/2$ the stationary regime is impossible, since the temperature increases without bound at internal points.

Problems of heat conduction with internal sources arise, for example, in heating of bodies by superhigh-frequency currents [1], in physical nuclear processes [2], and others. Similar nonstationary problems in the classical formulation for geometrically one-dimensional bodies have been considered in [3, 4]. It is very difficult to obtain exact analytical solutions in the case of specific engineering parts having a complex geometry [5]. One is able to do this only in the case of a classical geometry of a body. Solutions obtained for an unbounded angular region enable one to describe to an extent the processes of heat conduction in bodies whose shape has angles.

Nonstationary Problems for an Unbounded Rectangular Region. Let it be necessary to find the solution of the heat-conduction equation with an initial condition at $t = 0$ and a boundary condition at the boundary Γ of a rectangular region Ω:

$$u_t = a^2 \Delta u + q(t, x, y)\,, \quad u\big|_{t=0} = f(x, y)\,, \quad u\big|_{\Gamma} = 0\,, \quad x \ge 0\,, \quad y \ge 0\,. \tag{1}$$

In the case we can employ the theorem on the product of orthogonal solutions [6] and can take the Green function in the form

$$G = \frac{1}{4a^2 \pi t}\left[\exp\left(-\frac{(x-x^*)^2}{4a^2 t}\right) - \exp\left(-\frac{(x+x^*)^2}{4a^2 t}\right)\right] \times \left[\exp\left(-\frac{(y-y^*)^2}{4a^2 t}\right) - \exp\left(-\frac{(y+y^*)^2}{4a^2 t}\right)\right]. \tag{2}$$

The solution of problem (1) where a heat source $q(t, x, y)$ is acting inside Ω is given by the integral formula

$$u = \int_0^\infty dx^* \int_0^\infty G(t, x, y, x^*, y^*)\, f(x^*, y^*)\, dy^* + \int_0^t d\tau \int_0^\infty dx^* \int_0^\infty G(t-\tau, x, y, x^*, y^*)\, q(\tau, x^*, y^*)\, dy^*\,. \tag{3}$$

If the initial condition and the internal source are constant

$$f(x, y) = T_0\,, \quad q(\tau, x, y) = q_0\,, \tag{4}$$

the solution (3) is substantially simplified. For this purpose we must introduce new integration variables α_1 and α_2:

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$$\alpha_1 = \frac{x^* - x}{2a\sqrt{t}}, \quad \alpha_2 = \frac{x^* + x}{2a\sqrt{t}}, \quad d\alpha_1 = \frac{dx^*}{2a\sqrt{t}}, \quad d\alpha_2 = \frac{dx^*}{2a\sqrt{t}}. \tag{5}$$

Then, using the transformations described in [4], we can express the integral over the variable x^* from (3) in terms of the error integral $\Phi(z)$:

$$\int_0^\infty \frac{1}{2a\sqrt{\pi t}} \left[\exp\left(-\frac{(x - x^*)^2}{4a^2 t}\right) - \exp\left(-\frac{(x + x^*)^2}{4a^2 t}\right) \right] dx^* = \Phi\left(\frac{x}{2a\sqrt{t}}\right), \tag{6}$$

$$\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-\alpha^2)\, d\alpha, \quad z = \frac{x}{2a\sqrt{t}}.$$

Analogously we transform the integral over the variable y^* in (3), and the solution of problem (1) takes the form

$$u = T_0\, \Phi\left(\frac{x}{2a\sqrt{t}}\right) \Phi\left(\frac{y}{2a\sqrt{t}}\right) + q_0 \int_0^t \Phi\left(\frac{x}{2a\sqrt{t - \tau}}\right) \Phi\left(\frac{y}{2a\sqrt{t - \tau}}\right) d\tau. \tag{7}$$

Of interest is the limit for u at $t \to \infty$, upon whose computation we can answer the question of whether there exists the stationary regime when a constant heat source q_0 is acting inside. The latter heats the body, and the opposite process of extraction of heat occurs at its boundary since a constant zero temperature is maintained at Γ all the time. When $t \to \infty$, we have $z \to 0$ and the first term in (7) tends to zero; therefore, we drop it at once. Then we have

$$\lim_{t \to \infty} u = q_0 \lim_{t \to \infty} \int_0^t \Phi\left(\frac{x}{2a\sqrt{t - \tau}}\right) \Phi\left(\frac{y}{2a\sqrt{t - \tau}}\right) d\tau. \tag{8}$$

The error integral $\Phi(z)$ is a monotonically increasing function; therefore, for the point of the right angle where $x \le y$ we have the inequality

$$\int_0^t \Phi\left(\frac{x}{2a\sqrt{t - \tau}}\right) \Phi\left(\frac{y}{2a\sqrt{t - \tau}}\right) d\tau \ge \int_0^t \Phi^2\left(\frac{x}{2a\sqrt{t - \tau}}\right) d\tau. \tag{9}$$

To evaluate the improper integral in (8) we introduce the renotation of the integration variable

$$\frac{x}{2a\sqrt{t - \tau}} = \beta, \quad t - \tau = \frac{x^2}{4a^2\beta^2}, \quad d\tau = \frac{x^2 d\beta}{2a^2\beta^3}. \tag{10}$$

Using (9) and (10), we evaluate the limit (8) by the inequality

$$\lim_{t \to \infty} u \ge q_0 \lim_{t \to \infty} \int_{\frac{x}{2a\sqrt{t}}}^{\infty} \Phi^2(\beta) \frac{x^2 d\beta}{2a^2\beta^3} = \frac{q_0 x^2}{2a^2} \left[\int_0^b \frac{1}{\beta^3} \Phi^2(\beta)\, d\beta + \int_b^\infty \frac{1}{\beta^3} \Phi^2(\beta)\, d\beta \right] > \frac{q_0 x^2}{2a^2} \int_0^b \frac{1}{\beta^3} \Phi^2(\beta)\, d\beta, \tag{11}$$

where by b we will mean a certain positive quantity less than 1, i.e., $0 < b < 1$. For $\Phi(\beta)$ we write the convergent Taylor series

$$\Phi(\beta) = \frac{2}{\sqrt{\pi}} \left(\beta - \frac{\beta^3}{3} + \frac{\beta^5}{5 \cdot 2!} - \ldots + (-1)^{n+1} \frac{\beta^{2n-1}}{(2n-1)(n-1)!} + \ldots \right). \tag{12}$$

The alternating series (12) for $0 \le \beta \le b < 1$ consists of monotonically decreasing terms; therefore, we have the following evaluation:

$$\Phi(\beta) > \frac{2}{\sqrt{\pi}}\left(\beta - \frac{\beta^3}{3}\right), \quad 0 \le \beta < 1\,, \quad \Phi^2(\beta) > \frac{4}{\pi}\left(\beta^2 - \frac{2}{3}\beta^4 + \frac{\beta^6}{9}\right). \tag{13}$$

Substituting (13) into (11), we obtain the inequality

$$\lim_{t\to\infty} u > \frac{4}{\pi}\frac{q_0 x^2}{2a^2}\int\limits_0^b \frac{\beta^2 - 2\beta^4/3 + \beta^6/9}{\beta^3}\,d\beta\,. \tag{14}$$

The integral in (14) diverges; this means that the integral of (8) also diverges. In just the same manner we can prove that the improper integral (8) will diverge for the points $y < x$. It follows that with increase in t because of the action of the constant heat source q_0 the temperature increases without bound despite the extraction of heat through the boundary inside the rectangular region; therefore, the steady-state thermal regime is impossible.

Nonstationary Problem for an Acute-Angled Region. We locate the origin of coordinates at the vertex of an angle and introduce two auxiliary variables ξ_1 and ξ_2:

$$\xi_i = x n_{ix} + y n_{iy}\,, \quad i = 1, 2\,, \tag{15}$$

where $\mathbf{n}_i = (n_{ix}, n_{iy})$ is the vector of the unit normal to the ith side of the angle (the vector is directed into the angular region Ω). The equations of the sides of the angle will be determined by the equalities

$$\xi_1 = 0\,, \quad \xi_2 = 0\,. \tag{16}$$

The angle of opening κ_0 of the region Ω is expressed in terms of the scalar product of the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$:

$$\kappa_0 = \pi - \arccos(\mathbf{n}_1\mathbf{n}_2)\,. \tag{17}$$

We consider the auxiliary problem resulting from (1) in the absence of a heat source, i.e., at $q = 0$, if we formulate it for the half-space $x \ge 0$ and assume that u depends on t and only one geometric coordinate x:

$$u_t = a^2 u_{xx}\,, \quad u\,\big|_{t=0} = f(x)\,, \quad u\,\big|_{x=0} = 0\,. \tag{18}$$

The solution of problem (18) is known [4] and it has the form

$$u = F(t, x) = \frac{1}{2a\sqrt{\pi t}}\int\limits_0^\infty\left[\exp\left(-\frac{(x - x^*)^2}{4a^2 t}\right) - \exp\left(-\frac{(x + x^*)^2}{4a^2 t}\right)\right] f(x^*)\,dx^*\,. \tag{19}$$

Using ξ_1 and ξ_2, we additionally introduce the variables η_{ij}:

$$\eta_{ij} = \alpha_i\xi_1 + \alpha_j\xi_2\,, \quad i \ne j\,, \quad (i, j) = 1, 2, \ldots\,. \tag{20}$$

If $\forall\alpha_j \ge 0$, for $\xi_i \ge 0$, where $\xi_i \in \Omega$, we obtain $\forall\eta_{ij} \ge 0$, i.e., the variables $\eta_{ij} \ge 0$ are nonnegative inside Ω. Next, we formally replace the variable x in $F(t, x)$ from (19) η_{ij}. The requirement that $F(t, \eta_{ij})$ satisfy the heat-conduction equation (1) at $q = 0$ is equivalent to the requirement that the vector $(\alpha_i\mathbf{n}_1 + \alpha_2\mathbf{n}_2)$ be unit, i.e.,

$$\alpha_i^2 + \alpha_j^2 + 2\alpha_i\alpha_j(\mathbf{n}_1\mathbf{n}_2) = 1\,. \tag{21}$$

If α_i is plotted on the x axis and α_j is plotted on the y axis, expression (21), in the coordinate system (α_i, α_j), determines the ellipse whose M_{ij} points enable one to construct particular solutions $F(t, \eta_{ij})$ of the one-dimensional heat-conduction equation. We take $\alpha_1 = 1$ and $\alpha_0 = 0$ as the initial point M_{10}, i.e., $M_{10}(1, 0)$. The next point M_{12} on the ellipse (21) will be found by moving from point M_{10} in parallel to the *y* axis; then we must move from point M_{12} in parallel to the *x* axis and so on, alternating motions in parallel to the *x* and *y* axes. In these motions, each time we will find new points M_{ij} on the ellipse (21). The coordinates of M_{ij} are determined by the coefficients α_i the algorithm of whose finding leads to the following recurrence formula:

$$\alpha_{i+1} = -\alpha_{i-1} - 2B\alpha_i \, , \quad B = \mathbf{n}_1 \mathbf{n}_2 = -\cos \kappa_0 \, . \tag{22}$$

Let the procedure of obtaining points M_{ij} on the ellipse (21) be finite and the last point have coordinates (0, 1). This condition can be fulfilled only in the case where the angle of opening κ_0 of the angular region is equal to

$$\kappa_0 = \pi / (2n - 1) \, , \quad n = 1, 2, \dots \, . \tag{23}$$

Thus, the number *n* is determined by the opening angle κ_0 from formula (23). If condition (23) is not fulfilled, the procedure of obtaining points M_{ij} on the ellipse (21) will be infinite and the method proposed below will be unsuitable.

Let the angle κ_0 be computed from formula (23). For such opening angles the solution of problem (1) at $q = 0$ can be represented by the sum of the functions $F(t, \eta_{ij})$:

$$u = F(t, \eta_{10}) - F(t, \eta_{12}) + F(t, \eta_{32}) - \dots - F(t, \eta_{2n-1,2n}) + F(t, \eta_{2n+1,2n}) \, . \tag{24}$$

Here by $F(t, \eta_{ij})$ we must mean the following integral expression:

$$F(t, \eta_{ij}) = \frac{1}{2a\sqrt{\pi t}} \int_0^{\infty} \left[\exp\left(-\frac{(\eta_{ij} - x^*)^2}{4a^2 t} \right) - \exp\left(-\frac{(\eta_{ij} + x^*)^2}{4a^2 t} \right) \right] f(x^*)\, dx^* \, . \tag{25}$$

To check the fulfillment of the boundary condition $u|_{\Gamma} = 0$ we should employ the properties of the variables η_{ij}:

$$\eta_{2n+1,2n} = \eta_{01} \, , \quad \eta_{2n-1,2n} = \eta_{21} \quad \text{etc.,} \tag{26}$$

and the property at the boundary Γ at $\xi_1 = 0$ or at $\xi_2 = 0$:

$$\eta_{ij}|_{\xi_1=0} = \eta_{0j} \, , \quad \eta_{ij}|_{\xi_2=0} = \eta_{i0} \, . \tag{27}$$

Then at $\xi_1 = 0$ or $\xi_2 = 0$ all the terms but one in the sum of (24) are mutually eliminated, with the result that

$$u|_{\Gamma} = F(t, 0) = 0 \, . \tag{28}$$

The function $F(t, x)$ from (19) possesses the limiting property [4]

$$\lim_{t \to 0+0} F(t, x) = f(x) \, .$$

Therefore, the sum from (24) will satisfy the following initial condition:

$$\lim_{t \to 0+0} u = f(\eta_{10}) - f(\eta_{12}) + f(\eta_{32}) - \dots - f(\eta_{2n-1,2n}) + f(\eta_{2n+1,2n}) \, . \tag{29}$$

It follows from (29) that the solution (24) obtained is particular, since it corresponds to the particular expression of the initial condition that is determined just by the function $f(\eta_{ij})$ dependent on one geometric coordinate. Superposition of (29), conversely, will depend on two coordinates, i.e., *x* and *y*. Of greatest interest is the more particular case where $f(\eta_{ij}) = 1$. Then for each *n* from (29) we will have

$$\lim_{t\to 0+0} u = 1 . \tag{30}$$

Here $F(t, x)$, similarly to the transformations (5) and (6), can be reduced to the form

$$F(t, x) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2a\sqrt{t}}} \exp(-\alpha^2)\, d\alpha = \Phi\left(\frac{x}{2a\sqrt{t}}\right). \tag{31}$$

If we denote by E the particular solution at $f(\eta_{ij}) = 1$, from (24) we will have (using (31))

$$E = \Phi\left(\frac{\eta_{10}}{2a\sqrt{t}}\right) - \Phi\left(\frac{\eta_{12}}{2a\sqrt{t}}\right) + \Phi\left(\frac{\eta_{32}}{2a\sqrt{t}}\right) - \ldots - \Phi\left(\frac{\eta_{2n-1,2n}}{2a\sqrt{t}}\right) + \Phi\left(\frac{\eta_{2n+1,2n}}{2a\sqrt{t}}\right), \tag{32}$$

where E in construction is the solution of the boundary-value problem

$$E_t = a^2 \Delta E , \quad E\big|_{\Gamma} = 0 , \qquad \lim_{t\to 0+0} E = 1 . \tag{33}$$

Using u from (24) and E from (32), we can construct the following solution of the nonhomogeneous heat-conduction equation with nonhomogeneous boundary conditions:

$$U = u(t, x, y) + \mu(t) + \int_0^t E(t-\tau, x, y)\,[q(\tau) - \dot{\mu}(\tau)]\, d\tau . \tag{34}$$

Here U is the solution of the problem for the region Ω with the opening angle κ_0 from (23):

$$U_t = a^2 \Delta U + q(t) , \quad U\big|_{\Gamma} = \mu(t) ,$$

$$\lim_{t\to 0+0} u = \mu(0) + f(\eta_{10}) - f(\eta_{12}) + f(\eta_{32}) - \ldots - f(\eta_{2n-1,2n}) + f(\eta_{2n+1,2n}) . \tag{35}$$

The heat source $q(t)$ and the boundary condition $\mu(t)$ in (34) and (35) are assumed to be dependent on just one variable, t.

Stationary Solution for the Angular Region. In this case, the angle of opening κ_0 of the region Ω will be considered to be arbitrary for the present. Let it be necessary to find the solution of the Poisson equation with the boundary conditions

$$\Delta u + q_0 = 0 , \quad u\big|_{\Gamma} = 0 . \tag{36}$$

In the stationary regime, the temperature at the internal points of Ω is determined by two opposite factors: whereas the heat release q_0 contributes to a temperature increase, the extraction of heat through the boundary Γ contributes to a temperature decrease since a zero temperature is maintained all the time at Γ. From an analysis of the exact stationary solution which will be obtained below, we can draw the following conclusion: if the opening angle is $\kappa_0 \geq \pi/2$, the heat release by the source q_0 predominates over the extraction of heat through the boundary Γ and the temperature at each internal point of the region Ω undoubtedly increases; therefore, the stationary regime is impossible. The stationary regime is possible just for acute angular regions when $\kappa_0 < \pi/2$.

Formulation of the boundary-value problems for unbounded regions is usually supplemented with the condition at infinity [7]. In our case it is obtained from the following considerations. When the distance from the boundaries of the angle is rather large and the influence of the boundaries is weak, the temperature of the points must increase because of heat release, i.e., the following condition must be fulfilled: if $q_0 > 0$, for $(\xi_1, \xi_2) \to \infty$ we have

$$u(\xi_1, \xi_2) > 0 . \tag{37}$$

According to the formulation of problem (36) and (37), the function u interchangeably depends on ξ_1 and ξ_2; consequently, these variables must equally be involved in the solution proposed, which will be sought in the form of a polynomial of the second degree in ξ_1 and ξ_2:

$$u = A_0\xi_1\xi_2 + A_1 (\xi_1^2 + \xi_2^2) + A_2 (\xi_1 + \xi_2) + A_3 . \tag{38}$$

Since grad ξ_1 = $\mathbf{n}_1$ and grad ξ_2 = $\mathbf{n}_2$, from (38) we find

$$\begin{aligned} &\text{grad}\, u = A_0 (\mathbf{n}_1 \xi_2 + \mathbf{n}_2 \xi_1) + 2A_1 (\mathbf{n}_1 \xi_1 + \mathbf{n}_2 \xi_2) + A_2 (\mathbf{n}_1 + \mathbf{n}_2) , \\ &\frac{\partial u}{\partial n_1} = \mathbf{n}_1\, \text{grad}\, u = A_0 (\xi_2 + B\xi_1) + 2A_1 (\xi_1 + B\xi_2) + A_2 (1 + B) . \end{aligned} \tag{39}$$

Now we compute from (39) the Laplace operator

$$\Delta u = \text{grad}\,(\text{grad}\, u) = 2A_0B + 4A_1 . \tag{40}$$

After the substitution of Δu from (40) into (36) we obtain

$$2A_0B + 4A_1 = - q_0 , \tag{41}$$

and from the boundary conditions, for example, at $\xi_1 = 0$, we arrive at the equation

$$A_1\xi_2^2 + A_2\xi_2 + A_3 = 0 , \tag{42}$$

whence we find

$$A_1 = A_2 = A_3 = 0 , \quad A_0 = - q_0/(2B) . \tag{43}$$

Thus, the solution of the Dirichlet problem (36) and (37) for the angular region has the following form:

$$u = \frac{q_0}{2 \cos \kappa_0}\, \xi_1\xi_2 . \tag{44}$$

The solution obtained is suitable only for the acute angle $\kappa_0 < \pi/2$, since function (44) does not exist at $\kappa_0 = \pi/2$ and condition (37) is violated when $\kappa_0 > \pi/2$. We have proved above the impossibility of the stationary temperature field at $\kappa_0 = \pi/2$. Therefore, we finally arrive at the conclusion that the stationary thermal regime is impossible at $\kappa_0 \geq \pi/2$ in the presence of a constant heat source in the angular region, and when $\kappa_0 < \pi/2$ the stationary solution has the form (44). In this case the lines of the levels u = const are hyperbolas each of which has angle sides $\xi_1 = 0$ and $\xi_2 = 0$ as its asymptotes.

Stationary Problem with Mixed Boundary Conditions. We write the boundary conditions for Eq. (36) as

$$\left.\left(\alpha u - \lambda \frac{\partial u}{\partial n}\right)\right|_{\Gamma} = b_0 . \tag{45}$$

The solution will be sought in the form (38) as previously. Substituting (38) into boundary condition (45) at $\xi_1 = 0$ (or at $\xi_2 = 0$), we obtain

$$\alpha (A_1\xi_2^2 + A_2\xi_2 + A_3) - \lambda [A_0\xi_2 + 2A_1B\xi_2 + A_2 (1 + B)] = b_0 . \tag{46}$$

Whence we will have three equations:

$$\alpha A_1 = 0 , \quad \alpha A_2 - \lambda (A_0 + 2BA_1) = 0 , \quad \alpha A_3 - \lambda (1 + B) A_2 = b_0 . \tag{47}$$

From system (47) in combination with (41) at $\alpha \neq 0$ and $B \neq 0$ we find A_i:

$$A_1 = 0 \ , \quad A_0 = -\frac{q_0}{2B} \ , \quad A_2 = -\frac{\lambda q_0}{2\alpha B} \ , \quad A_3 = \frac{b_0}{\alpha} - (1 + B)\frac{\lambda^2 q_0}{2\alpha^2 B} \ . \tag{48}$$

Substituting A_i from (48) into (38), we obtain the sought solution:

$$u = -\frac{q_0}{2B}\xi_1\xi_2 - \frac{\lambda q_0}{2\alpha B}(\xi_1 + \xi_2) + \frac{b_0}{\alpha} - \left(1 + \frac{1}{B}\right)\frac{\lambda^2 q_0}{2\alpha^2} \ . \tag{49}$$

If the opening angle is $\kappa_0 = \pi/2$, then $B = 0$ and the solution (49) does not exist. When $\kappa_0 > \pi/2$, we have $0 < B < 1$ and then, for rather large ξ_1 and ξ_2, we will have $u < 0$, i.e., condition (37) will be violated. The obtained solution (49) exists only for the acute angles $\kappa_0 < \pi/2$; otherwise, the stationary regime with mixed-type boundary conditions is impossible similarly to the previous case of the Dirichlet problem.

The quantity u takes the lowest value at the vertex of the angle:

$$u\big|_{\xi_1=\xi_2=0} = \frac{b_0}{\alpha} - \left(1 + \frac{1}{B}\right)\frac{\lambda^2 q_0}{2\alpha^2} = u_{00} \ . \tag{50}$$

On the side $\xi_2 = 0$, we have

$$u\big|_{\xi_2=0} = u_{00} - \frac{\lambda q_0}{2\alpha B}\xi_1 \ , \tag{51}$$

i.e., with distance from the vertex of the angle along the side $\xi_2 = 0$ the function u increases by the linear law in relation to the distance to the vertex.

NOTATION

Ω, region of the angle; κ_0, its opening angle; Γ, boundary of the angular region; t, time; (x, y), Cartesian coordinates; Δ, Laplace operator; u, temperature of the points of the region Ω; a^2, thermal diffusivity; $f(x, y)$, initial temperature; q, heat flux; G, Green function; (x^*, y^*), geometric variables of integration; τ, time variable of integration; T_0, constant initial temperature; q_0, constant internal heat source; $\Phi(z)$, error integral — Laplace function; $\mathbf{n}_1$ and $\mathbf{n}_2$, unit normals to the sides of the angle Ω; ξ_i and η_{ij}, auxiliary geometric variables; α_i and α_j, coefficients for construction of the variables η_{ij}; $\mu(t)$, temperature at the boundary; A_0–A_3, constant coefficients of the stationary solution; α, heat-transfer coefficient; λ, thermal conductivity; b_0, constant in a mixed-type boundary condition.

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